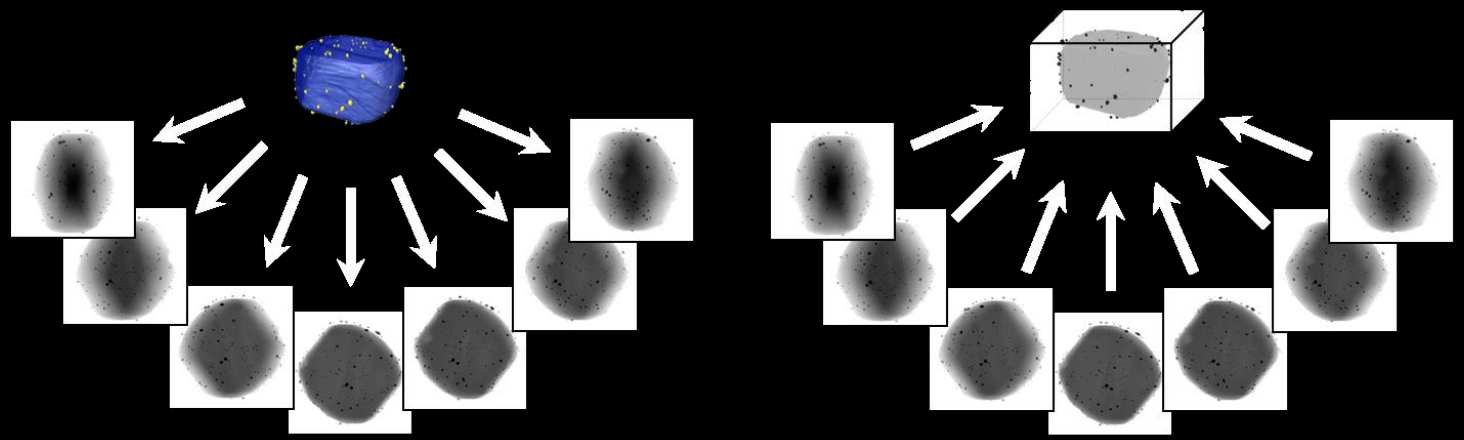
# Tomographic Image Reconstruction by Total-Variation Minimization

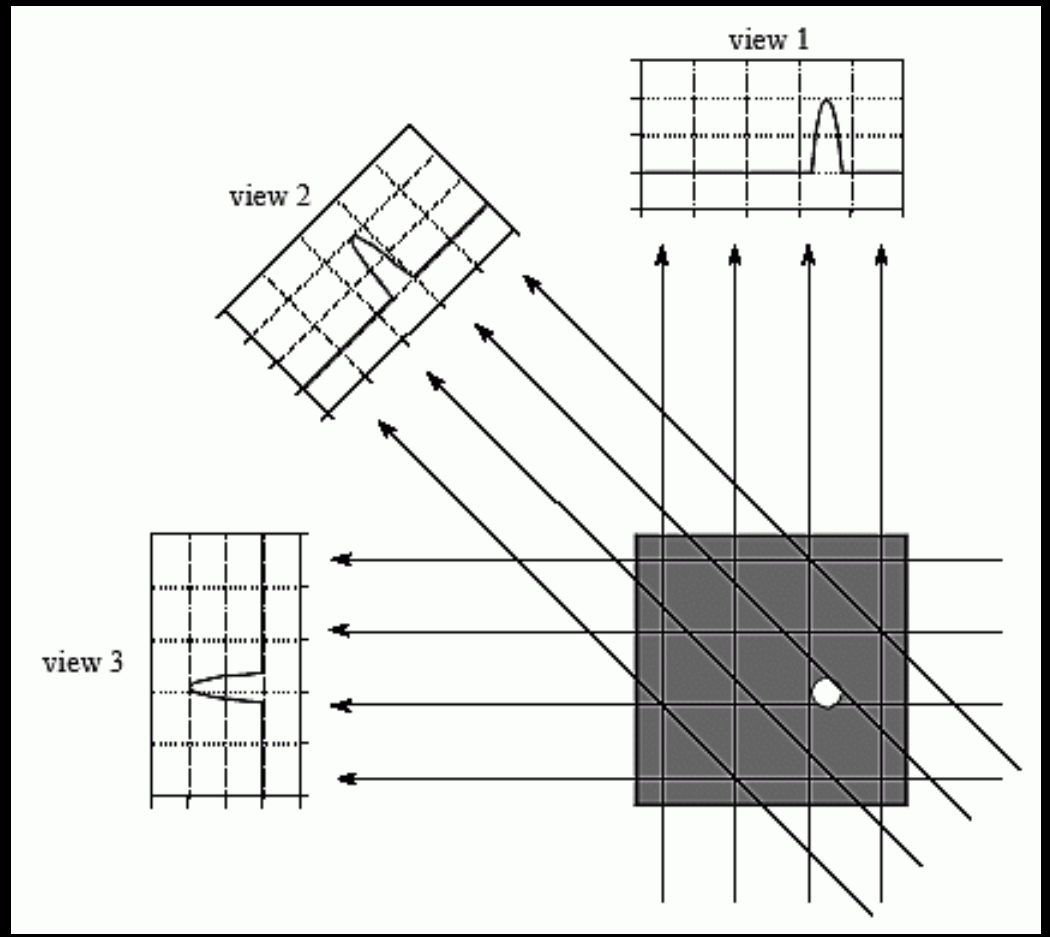
Zhifei Zhang

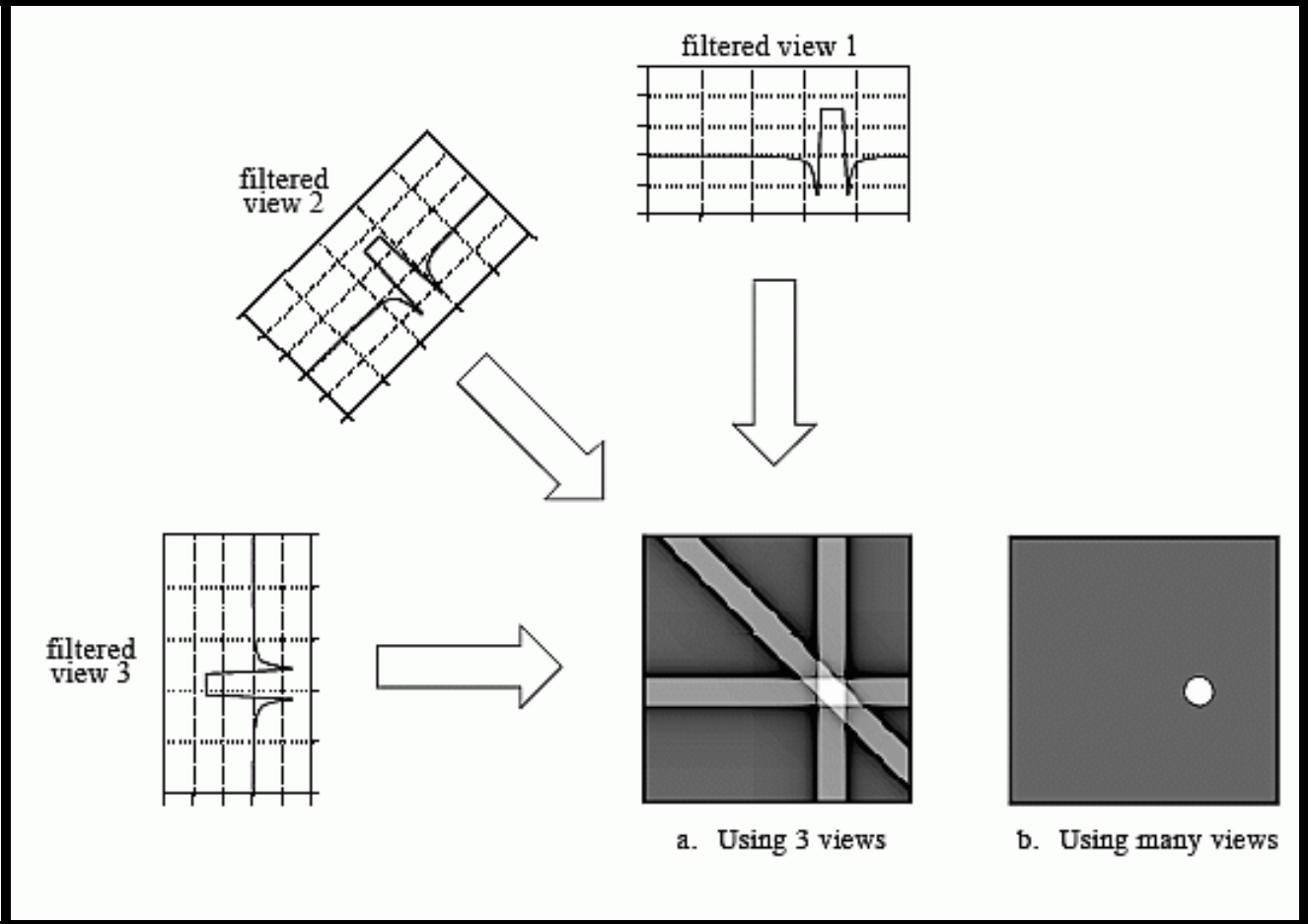


# Outline

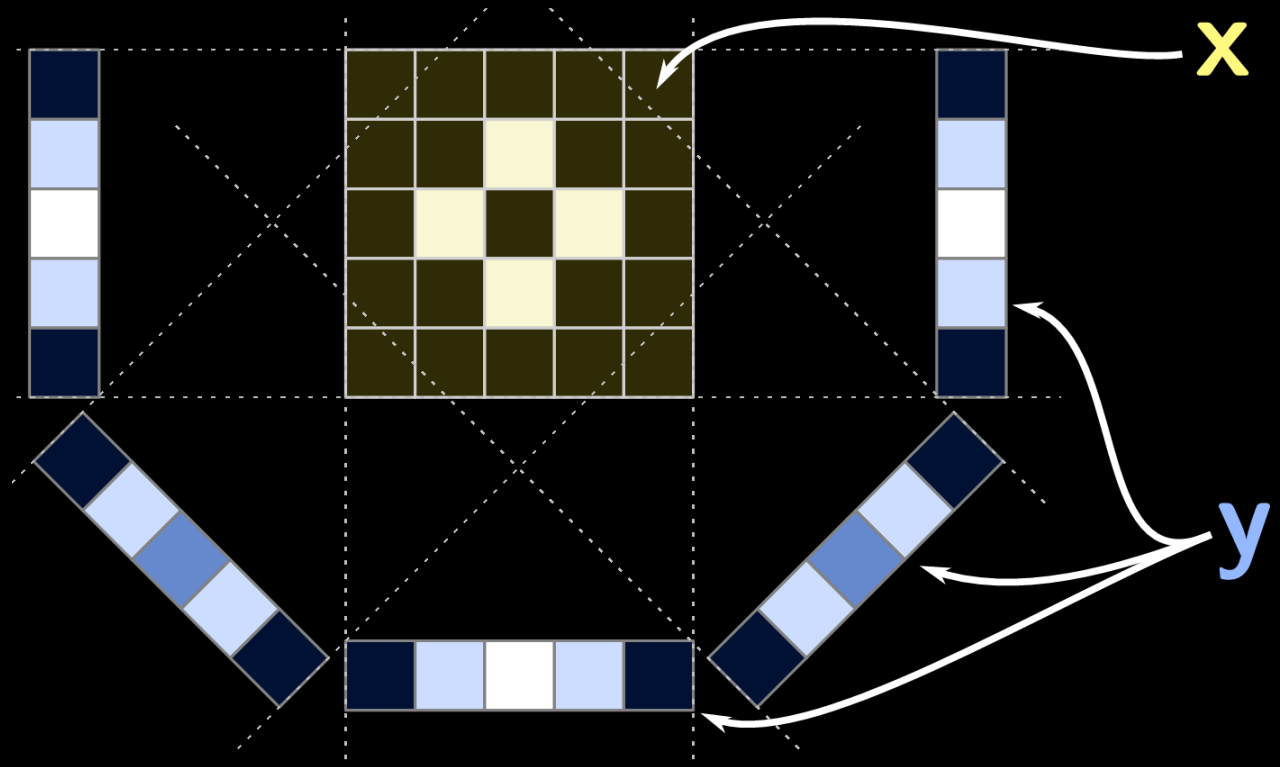
1. Learned in class
2. Total variation
3. Comparison

# Filtered Back Projection (FBP)



# Least Square (LS)




$$Ax = y$$

$$\min_{x>0} \frac{1}{2} \|Ax - y\|^2$$

**Find x knowing A and y**

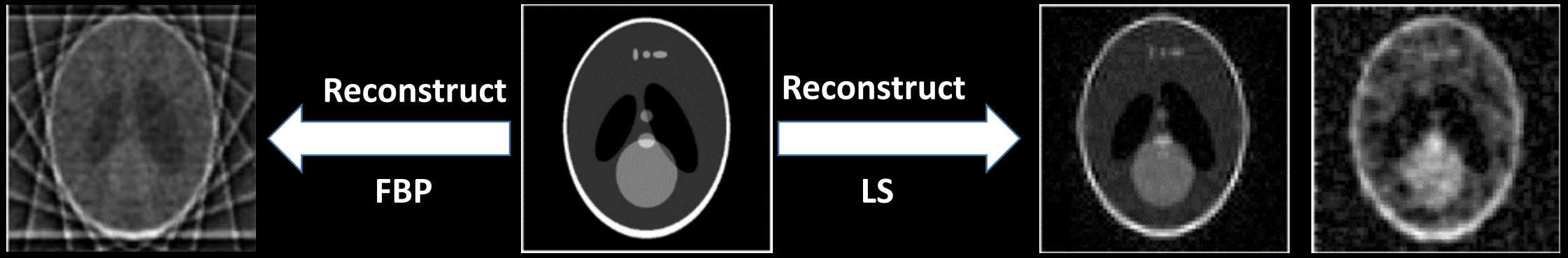
# Drawbacks of FBP and LS

## FBP:

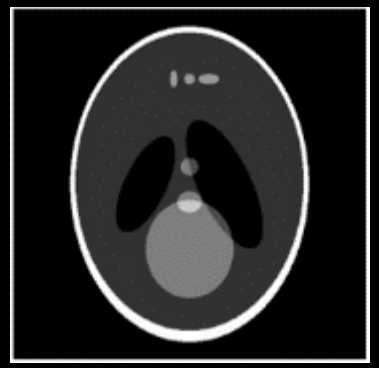
- Need enough projections
- Sharp edge or noise

## LS:

- Fit noise (over-fitting)
- Sensitive to outliers

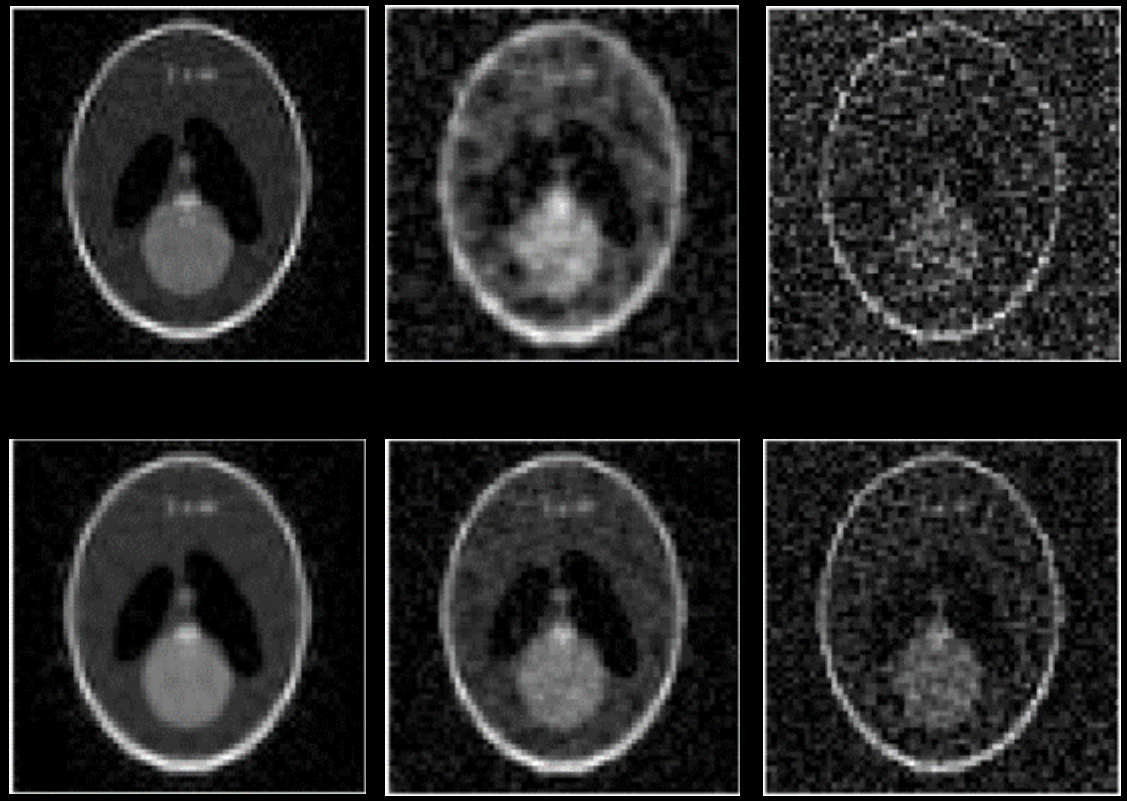


# What can Total Variation Achieve?



Original

Reconstruct



# What is Total Variation (TV)?

$$TV = \sum_{i,j=1}^{n} |I_{ij} - I_{i,j+1}| + |I_{ij} - I_{i+1,j}|$$

TV

$$\begin{pmatrix} 124 & 100 & \cdots \\ 69 & \cdots & \cdots \\ \vdots & \cdots & \cdots \end{pmatrix}_{n\times n}$$

$$\begin{pmatrix} 124 & 100 & \cdots \\ 69 & \cdots & \cdots \\ \vdots & \cdots & \cdots \end{pmatrix}_{n\times n}$$

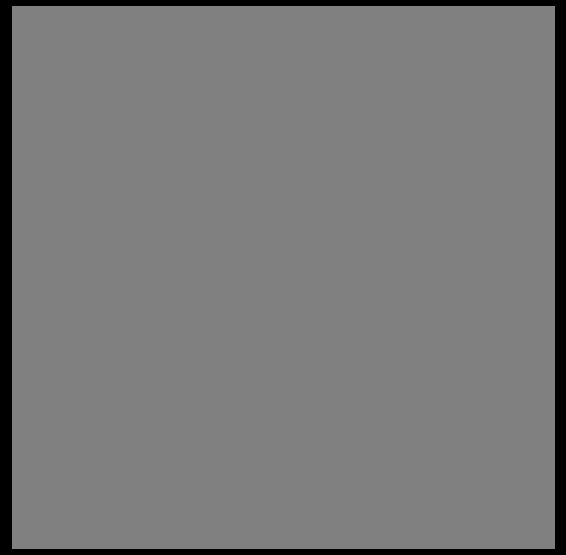
# What is Total Variation (TV)?

## If let $TV$ approach zero

Original

$TV$ = high

$TV$ = low



$TV$ = 0

# Total-Variation Minimization

WANTED

$$\arg\min_{x>0} \frac{1}{2}\|Ax - y\|^2 + \lambda \cdot TV \quad (\lambda > 0)$$

System model

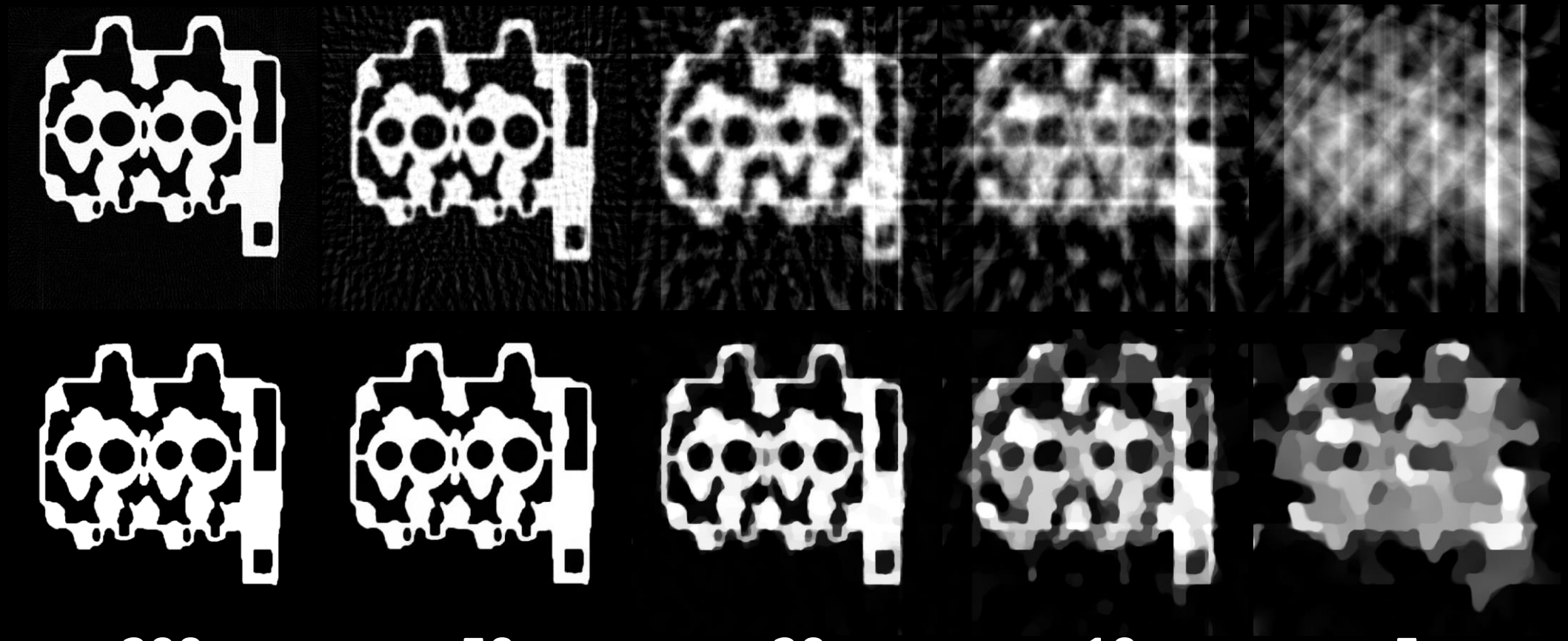
Projection

Penalty parameter

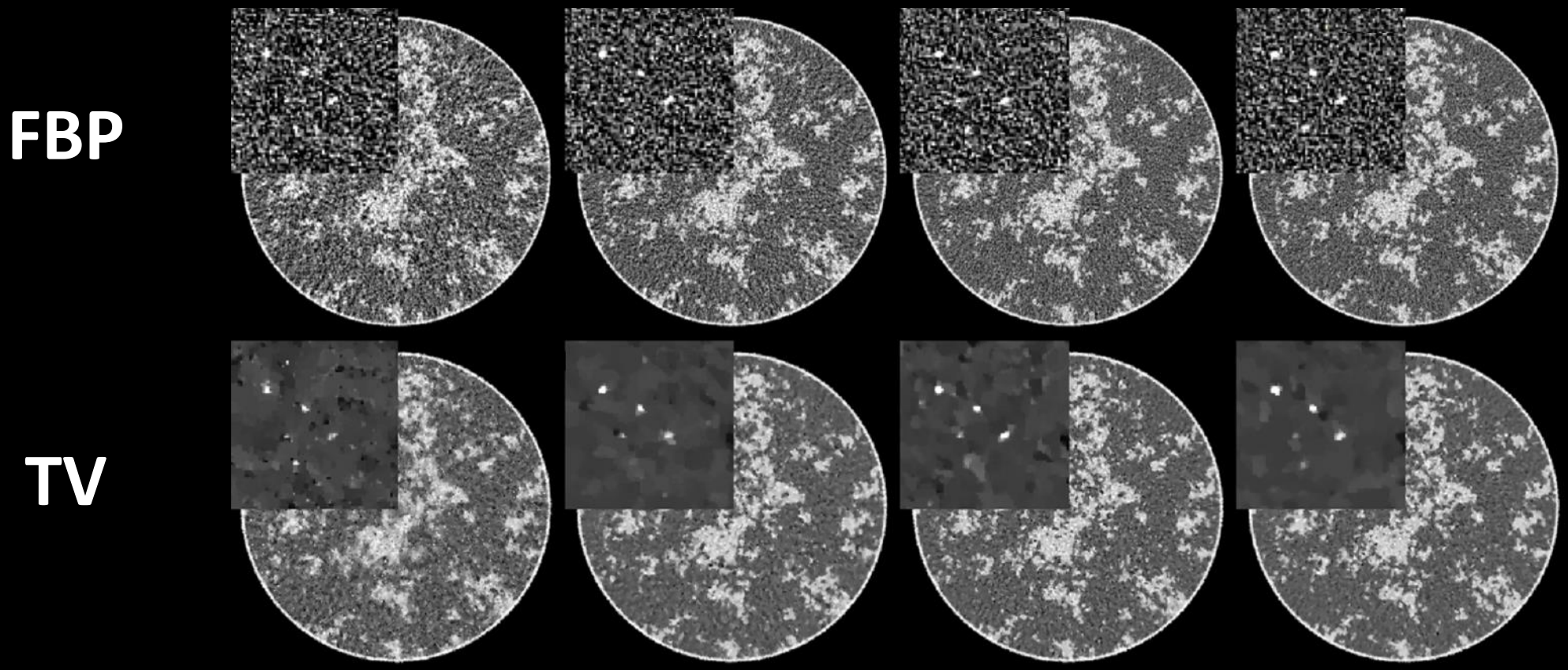
$$TV = \sum_{i=1}^{m}\sum_{j=1}^{n} \|\nabla x_{ij}\|_2$$

$\nabla x_{ij}$ denotes gradient approximation for pixel *ij*

# Comparison (Less Projections)



# Comparison (Less Projections)



# Conclusion

- Image reconstruction by TV minimization
- Make image more smooth and edge clear
- Robust to noise and need less projections
- It may be tricky to set the TV parameter $\lambda$

  (get balance between removing noise and preserving details)

# Thank You!

**References**


- Hansen, Per Christian, and Jakob Heide Jørgensen. "Total Variation and Tomographic Imaging from Projections." *36th Conference of the Dutch-Flemish Numerical Analysis Communities*.
- Dahl, Joachim, et al. "Algorithms and software for total variation image reconstruction via first-order methods." *Numerical Algorithms* 53.1 (2010): 67-92.